Physics 11 Fall 2012
Practice Problems 4 - Solutions

1. Under what conditions can all the initial kinetic energy of an isolated system consisting of two colliding objects be lost in a collision? Explain how this result can be, and yet the momentum of the system can be conserved?

Solution

If the collision is perfectly inelastic, then the colliding particles stick together. If the two particles have the same momentum, and are traveling in opposite directions, then when they collide the two particles’ momentum cancel each other out. The particles stop, and so they don’t have any kinetic energy, but the momentum is still conserved.
2. Bored, a boy shoots his pellet gun at a piece of cheese that sits on a massive block of ice. On one particular shot, his 1.2 g pellet gets stuck in the cheese, causing it to slide 25 cm before coming to a stop. If the muzzle velocity of the gun is known to be 65 m/s and the cheese has a mass of 120 g, what is the coefficient of friction between the cheese and the ice?

Solution

Let’s start by looking at the momentum. Since the cheese isn’t moving, initially, the initial momentum is just that of the pellet, \( p_i = p_{\text{pellet}} \). If the mass of the pellet is \( m \), and it has an initial velocity \( v \), then \( p_i = mv \). After the collision, the pellet is stuck in the cheese. If the cheese has a mass \( M \), and if the system has a final velocity \( V \), then the final momentum is \( p_f = (m + M) V \). Solving for the final velocity gives

\[
V = \left( \frac{m}{m + M} \right) v.
\]

So, the combined system has an initial kinetic energy of \( KE = \frac{1}{2} (m + M) V^2 = \frac{m^2}{2(m + M)} v^2 \). After the cheese/pellet system has come to rest, its kinetic energy is zero. The energy lost has to go into the work done by friction, which is \( W = \mu_k F_N d = \mu_k (m + M) gd \), where \( d \) is the distance the system slides. So, setting this equal to the kinetic energy and solving for the coefficient we find

\[
\mu_k = \frac{m^2 v^2}{2 (m + M)^2 gd}.
\]

Plugging in numbers gives

\[
\mu_k = \frac{m^2 v^2}{2 (m + M)^2 gd} = \frac{0.0012^2 \times 65^2}{2 (0.1212)^2 \times 9.8 \times 0.25} = 0.085
\]
3. Most geologists believe that the dinosaurs became extinct 65 million years ago when a large comet or asteroid struck the earth, throwing up so much dust that the sun was blocked out for a period of many months. Suppose an asteroid with a diameter of 2.0 km and a mass of $1.0 \times 10^{13}$ kg hits the earth with an impact speed of $4.0 \times 10^4$ m/s.

(a) What is the earth’s recoil speed after such a collision? (Use a reference frame in which the earth was initially at rest.)

(b) What percentage is this of the earth’s speed around the sun?

Solution

(a) Initially, in our reference frame, the earth is at rest, while the asteroid is moving towards it, so $p_{\text{Earth}} = 0$, $p_{\text{asteroid}} = mv_i$, where $m$ is the mass of the asteroid. After the collision, the asteroid and Earth are one system of a total mass $m + M$, where $M$ is the mass of the earth, and a final velocity $V$, which is the recoil speed of the earth. So, the final momentum of the total system is $p_f = (m + M)V$. Conservation of momentum says $p_i = p_f$, and so the final velocity is

$$V = \frac{m}{m + M}v.$$ 

The mass of the earth is $5.97 \times 10^{24}$ kg, and so we have

$$V = \frac{m}{m + M}v = \left[\frac{10^{13}}{10^{13} + 5.97 \times 10^{24}}\right] \times 4 \times 10^4 = 6.7 \times 10^{-8} \text{ m/s}.$$ 

(b) The earth has an orbital radius of about $150 \times 10^9$ meters, and it takes one year, or about $\pi \times 10^7$ seconds to go around, so the velocity is $v = d/T = 2\pi r/T \approx \frac{2\pi (150 \times 10^9)}{\pi \times 10^7} = 30000$ m/s, or about 30 km/s. So, the ratio of the recoil speed to the orbital speed is $6.7 \times 10^{-8}/3 \times 10^4 \approx 2 \times 10^{-12}$, or about $2 \times 10^{-10}$ %, which is tiny!
4. A proton of mass \( m \) is moving with initial speed \( v_0 \) directly toward the center of an \( \alpha \) particle of mass \( 4m \), which is initially at rest. Both particles carry positive charge, so they repel each other. (The repulsive forces are sufficient to prevent the two particles from coming into direct contact.) Find the speed \( v_\alpha \) of the \( \alpha \) particle

(a) when the distance between the two particles is a minimum, and

(b) later when the two particles are far apart.

**Solution**

(a) When the particles are as close as they can get, the relative velocity between them is zero. So, they are both moving at the same speed, \( V \). From the conservation of momentum, \( mv_0 = (m + 4m)V = 5mV \). So, we can see immediately that \( V = v_\alpha = \frac{1}{5}v_0 \).

(b) When the particles are far apart, they aren’t interacting any more, and their potential energy is zero - they only have kinetic energy. The kinetic energy of the initial proton was \( \frac{1}{2}mv_0^2 \), while the final kinetic energy is \( \frac{1}{2}mv_f^2 + \frac{1}{2}(4m)v_\alpha^2 \). Conservation of energy sets the two equal.

Momentum is also conserved in this reaction. The initial momentum is \( mv_0 \), while the final momentum is \( -mv_f + 4mv_\alpha \), where the negative term is because the proton is reflected backwards. So, \( v_f = 4v_\alpha - v_0 \). Plugging this into the conservation of energy equation gives

\[
\frac{v_0^2}{2} = \frac{v_f^2}{2} + 4v_\alpha^2
\]

\[
= (4v_\alpha - v_0)^2 + 4v_\alpha^2
\]

\[
= 20v_\alpha^2 - 8v_0v_\alpha + v_0^2.
\]

Solving for \( v_\alpha \) gives

\[
v_\alpha = \frac{2}{5}v_0.
\]
5. Show that in a one-dimensional elastic collision, if the mass and velocity of object 1 are $m_1$ and $v_{1i}$, and if the mass and velocity of object 2 are $m_2$ and $v_{2i}$, then their final velocities $v_{1f}$ and $v_{2f}$ are given by

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$ 

Solution

Momentum is conserved in a collision, and so

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$ 

Furthermore, in an elastic collision the kinetic energy is also conserved,

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2.$$ 

This gives us two equations with two unknowns ($v_{1f}$ and $v_{2f}$), and so we can solve them; it’s just algebra from here. Solve the momentum equation for $v_{2f}$ to find

$$v_{2f} - v_{2i} = \frac{m_1}{m_2} (v_{1i} - v_{1f}).$$

We can rewrite the kinetic energy expression as

$$m_1 (v_{1f}^2 - v_{1i}^2) = m_2 (v_{2i}^2 - v_{2f}^2) = m_1 (v_{1f} - v_{1i}) (v_{1f} + v_{1i}) = m_2 (v_{2i} - v_{2f}) (v_{2f} + v_{2i}).$$

But, from the momentum equation $m_1 (v_{1f} - v_{1i}) = m_2 (v_{2i} - v_{2f})$. So, the kinetic energy expression becomes

$$v_{1f} + v_{1i} = v_{2f} + v_{2i}.$$ 

Now, solving the momentum equation for $v_{2f}$ gives $v_{2f} = v_{2i} + \frac{m_1}{m_2} (v_{1i} - v_{1f})$. Plugging this into the above equation gives

$$v_{1f} + v_{1i} = 2v_{2i} + \frac{m_1}{m_2} (v_{1i} - v_{1f}).$$

Rewriting gives

$$\left(1 + \frac{m_1}{m_2}\right) v_{1f} = 2v_{2i} + \left(\frac{m_1}{m_2} - 1\right) v_{1i} \Rightarrow v_{1f} = \frac{2v_{2i}}{1 + m_2/m_1} + \frac{m_1/m_2 - 1}{m_1/m_2 + 1} v_{1i},$$

or

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + 2\frac{m_2}{m_1 + m_2} v_{2i}.$$ 

Now, since $v_{2f} = v_{1f} + v_{1i} - v_{2i}$, we can easily solve for $v_{2f}$,

$$v_{2f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + 2\frac{m_2 v_{2i}}{m_1 + m_2} + v_{1i} - v_{2i} = \frac{1}{m_1 + m_2} v_{1i} \left(m_1 - m_2 + m_1 + m_2\right) + \frac{2m_1}{m_1 + m_2} (2m_2 - m_1 - m_2)$$

$$= \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$ 

That solves the problem.
6. A bullet of mass $m_1$ is fired horizontally with a speed $v_0$ into the bob of a ballistic pendulum of mass $m_2$. The pendulum consists of a bob attached to one end of a very light rod of length $L$. The rod is free to rotate about a horizontal axis through its other end. The bullet is stopped in the bob. Find the minimum $v_0$ such that the bob will swing through a complete circle.

Solution

The initial momentum of the bullet is $m_1 v_0$. When the bullet is embedded in the bob, then it picks up a momentum $(m_1 + m_2) V$, where $V$ is the speed of the composite system. Solving the momentum expression gives $V = \frac{m_1}{m_1 + m_2} v_0$. So, the initial kinetic energy of the system is $\frac{1}{2} (m_1 + m_2) V^2 = \frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_0^2$.

As the bob swings around it changes this kinetic energy into potential energy, gaining $(m_1 + m_2) gh$ at a height $h$. At the top of the circle, the height of the bob is $2L$, and so the potential energy is $2(m_1 + m_2)gL$. If the pendulum just reaches this height, then the kinetic energy is just equal to the potential energy. Thus,

$$\frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_0^2 = 2 (m_1 + m_2) gL.$$

Solving for the initial velocity gives

$$v_0 = 2 \frac{m_1 + m_2}{m_1} \sqrt{gL} = 2 \left( 1 + \frac{m_2}{m_1} \right) \sqrt{gL}.$$
7. The light isotope, $^5\text{Li}$, of lithium is unstable and breaks up spontaneously into a proton and an $\alpha$ particle. In this process, $3.15 \times 10^{-13}$ J of energy are released, appearing as the kinetic energy of the two decay products. Determine the velocities of the proton and the $\alpha$ particle that arise from the decay of a $^5\text{Li}$ nucleus at rest. (Note: The masses of the proton and the alpha particle are $m_p = 1.67 \times 10^{-27}$ kg and $m_\alpha = 4m_p = 6.64 \times 10^{-27}$ kg.)

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**Solution**

The nucleus was initially at rest, so it’s initial momentum is zero. After the decay, the products fly out, back to back, conserving momentum. So, $p_p = m_p v_p = -m_\alpha v_\alpha = -p_\alpha$. The kinetic energy is not conserved, but comes from the decay process. If we call the energy released $E$, then $E = KE_p + KE_\alpha = \frac{1}{2}m_p v_p^2 + \frac{1}{2}m_\alpha v_\alpha^2$. Solving the momentum expression for $v_p = -\frac{m_\alpha}{m_p} v_\alpha$, and plugging the result back into the energy expression gives

$$E = \frac{1}{2} \left(1 + \frac{m_\alpha}{m_p}\right) m_\alpha v_\alpha^2.$$

So, we can solve for the velocity of the $\alpha$ particle,

$$v_\alpha = \sqrt{\frac{2m_p}{m_\alpha} \frac{E}{m_p + m_\alpha}}.$$

Since $v_p = -\frac{m_\alpha}{m_p} v_\alpha$, then

$$v_p = -\sqrt{\frac{2m_\alpha}{m_p} \frac{E}{m_p + m_\alpha}}.$$

Plugging in the numbers gives

$$v_\alpha = \sqrt{\frac{2m_p}{m_\alpha} \frac{3.15 \times 10^{-13}}{m_p + m_\alpha}} = \sqrt{\frac{2}{4} \frac{3.15 \times 10^{-13}}{1.67 + 6.64} \times 10^{-27}} = 4.36 \times 10^6 \text{ m/s},$$

while

$$v_p = -\sqrt{\frac{2m_\alpha}{m_p} \frac{3.15 \times 10^{-13}}{m_p + m_\alpha}} = \sqrt{\frac{2 \times 4}{(1.67 + 6.64) \times 10^{-27}}} = 1.74 \times 10^7 \text{ m/s}.$$
8. In a pool game, the cue ball, which has an initial speed of 5.0 m/s, makes an elastic collision with the eight ball, which is initially at rest. After the collision, the eight ball moves at an angle of 30° to the right of the original direction of the cue ball. Assume that the balls have equal mass.

(a) Find the direction of motion of the cue ball immediately after the collision.
(b) Find the speed of each ball immediately after the collision.

Solution

(a) The collision conserves momentum, but this time the collision takes place in two dimensions. So, we can write the momentum conservation equation as a vector equation,

\[ \vec{p_i} = m_{\text{cue}}\vec{v_i} = m_{\text{cue}}\vec{v_f} + m_8\vec{V_f}, \]

where \( \vec{V} \) is the velocity vector of the eight ball. Momentum is conserved in both the \( x \) and \( y \) directions, and so, if we assume that the cue ball was originally moving along the \( y \) direction (to make the angles easier), and recalling that the masses are equal we have

Along \( x \)
\[ 0 = -v_f \cos \theta + V_f \cos \alpha \]
Along \( y \)
\[ v_i = v_f \sin \theta + V_f \sin \alpha. \]

Here we have defined the angle the cue ball moves to be \( \theta \), while the angle the eight ball moves is \( \alpha \), and the first term along \( x \) is negative since the eight ball moves to the right. Now, square the first expression to find

\[ 0 = v_f^2 \cos^2 \theta - 2v_f V_f \cos \theta \cos \alpha + V_f^2 \cos^2 \alpha, \]

while squaring the second gives

\[ v_i^2 = v_f^2 \sin^2 \theta + 2v_f V_f \sin \theta \sin \alpha + V_f^2 \sin^2 \alpha. \]

Adding this result to the last one gives

\[ v_i^2 = v_f^2 + V_f^2 + 2v_f V_f \cos (\alpha + \theta), \]

where we have used a couple of trign identities.

Since this is an elastic collision, the kinetic energy is conserved,

\[ \frac{1}{2}m_{\text{cue}}v_i^2 = \frac{1}{2}m_{\text{cue}}v_f^2 + \frac{1}{2}m_8V_f^2 \Rightarrow v_i^2 = v_f^2 + V_f^2, \]

since the masses of the balls are equal. Comparing this to our last result we see that \( \cos (\alpha + \theta) = 0 \), and so \( \alpha + \theta = 90° \). Since \( \alpha = 30° \), then \( \theta = 60° \).
(b) We want $v_f$ and $V_f$. Now, from the momentum equation, $v_f = V_f \frac{\cos \alpha}{\cos \theta}$. But $v_f^2 = V_f^2 - v_i^2$, and so we have $V_f^2 \frac{\cos^2 \alpha}{\cos^2 \theta} = v_i^2 + V_f^2$, so

$$V_f = \frac{v_i \cos \theta}{\sqrt{\cos^2 \theta + \cos^2 \alpha}}$$

while

$$v_f = \frac{v_i \cos \alpha}{\sqrt{\cos^2 \theta + \cos^2 \alpha}}.$$

So, plugging in the numbers gives

$$V_f = \frac{5 \cos 60}{\sqrt{\cos^2 60 + \cos^2 30}} = 2.5 \text{ m/s},$$

and

$$v_f = \frac{5 \cos 30}{\sqrt{\cos^2 60 + \cos^2 30}} = 4.33 \text{ m/s}.$$
9. In the “slingshot effect,” the transfer of energy in an elastic collision is used to boost the energy of a space probe so that it can escape from the solar system. All speeds are relative to an inertial frame in which the center of the Sun remains at rest. The figure shows a space probe moving at 10.4 km/s toward Saturn, which is moving at 9.6 km/s toward the probe. Because of the gravitational attraction between Saturn and the probe, the probe swings around Saturn and heads back in the opposite direction with speed \( v_f \).

(a) Assuming this collision to be a one-dimensional elastic collision with the mass of Saturn much much greater than that of the probe, find \( v_f \).

(b) By what factor is the kinetic energy of the probe increased? Where does this energy come from?

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**Solution**

(a) This is just a one-dimensional elastic collision, and so if we look back at problem 5, we found that

\[
 v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i},
\]

where \( m_1 \) is the probe, and \( m_2 \) is Saturn. Since \( m_2 \gg m_1 \), then

\[
 v_{1f} \approx -v_{1i} + 2v_{2i} = -10.4 + 2 \times (-9.6) = -30 \text{ km/s}.
\]

(b) The final kinetic energy energy is \( \frac{1}{2}m_1v_f^2 \), while the initial kinetic energy is \( \frac{1}{2}mv_i^2 \). The ratio of the two is

\[
 \frac{KE_f}{KE_i} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \left( \frac{v_f}{v_i} \right)^2 = \left( \frac{30}{10.4} \right)^2 \approx 8.3.
\]

So the kinetic energy is roughly eight times bigger than it was originally. This energy comes at the expense of some of the momentum of Saturn. However, since Saturn is much more massive than the probe, there’s very little effect the motion of Saturn, slowing down by a tiny bit.
10. A neutron of mass $m$ makes an elastic head-on collision with a stationary nucleus of mass $M$.

(a) Show that the kinetic energy of the nucleus after the collision is given by $K_{\text{nucleus}} = \left[\frac{4mM}{(m + M)^2}\right] K_n$, where $K_n$ is the initial kinetic energy of the neutron.

(b) Show that the fractional change in the kinetic energy of the neutron is given by

$$\frac{\Delta K_n}{K_n} = -\frac{4(m/M)}{1 + [m/M]^2}.$$  

(c) Show that this expression gives plausible results both if $m \ll M$ and if $m = M$.

What is the best stationary nucleus for the neutron to collide head-on with if the objective is to produce a maximum loss in kinetic energy of the neutron?

Solution

(a) The elastic collision conserves both momentum and kinetic energy. Since the nucleus is initially stationary, it’s initial momentum is zero, as is its initial kinetic energy. Once again we can go back to problem 5. So, if we write “$v$” for the neutron’s initial velocity, and “$V$” for the final velocity of the nucleus, then the final velocity of the nucleus is

$$V = \frac{2m}{m + M} v.$$  

Now the kinetic energy of the nucleus is $\frac{1}{2}MV^2$, or

$$K_{\text{E}} = \frac{1}{2}MV^2 = \frac{1}{2}M \left(\frac{4m^2}{(m + M)^2}\right) v^2.$$  

We can rewrite this as

$$K_{\text{E}} = \frac{1}{2}mv^2 \left[\frac{4mM}{(m + M)^2}\right].$$  

But, $\frac{1}{2}mv^2 = KE_n$, the kinetic energy of the neutron. So, we have

$$K_{\text{E}} = \frac{4mM}{(m + M)^2} KE_n.$$  

(b) The fractional change is just $\Delta KE_n/KE_n = \frac{1}{KE_n} (KE_{nf} - KE_{ni})$. Since this is an elastic collision, the total kinetic energy of the system is conserved. So, any change in the neutron’s kinetic energy had to go to the kinetic energy of the nucleus, $KE_{nf} - KE_{ni} = -KE_N$, since the final energy is lower than the initial energy. Thus,

$$\frac{\Delta KE_n}{KE_n} = -\frac{KE_N}{KE_n} = -\frac{1}{KE_n} \frac{4mM}{(m + M)^2} KE_n.$$  

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Thus, we find
\[
\frac{\Delta KE_n}{KE_n} = -\frac{4mM}{(m + M)^2} = -\frac{4(m/M)}{(1 + [m/M])^2}.
\]

(c) If \(m \ll M\), then the above expression becomes
\[
\frac{\Delta KE}{KE} \approx -4 \frac{m}{M} \to 0.
\]

This makes sense - the nucleus would be so heavy that the neutron would hardly nudge it. If the masses are equal (basically you’d be hitting a hydrogen nucleus with the neutron), then you would find
\[
\frac{\Delta KE}{KE} = -\frac{4}{2^2} = -1.
\]

In this case the neutron transfers it’s entire kinetic energy to the nucleus. So, the best nucleus to use would be hydrogen.