Physics 11 Fall 2012
Practice Problems 2 - Solutions

1. True or false (ignore any effects due to air resistance):

   (a) When a projectile is fired horizontally, it takes the same amount of time to reach
       the ground as an identical projectile dropped from rest from the same height.
   (b) When a projectile is fired from a certain height at an upward angle, it takes longer
       to reach the ground than does an identical projectile dropped from rest from the
       same height.
   (c) When a projectile is fired horizontally from a certain height, it has a higher speed
       upon reaching the ground than does an identical projectile dropped from rest
       from the same height.

Solution

(a) True. The rate at which the objects fall is independent of their horizontal velocity.

(b) True. The projectile fired up starts heading up first, before it starts heading
    toward the ground. So, it will take longer to hit the ground than an object just
    dropped from rest.

(c) True. The object just dropped picks up speed from gravitational acceleration. The
    fired projectile also picks up this speed, but it also had the initial velocity in
    the horizontal direction. The final speed includes both these contributions, and
    so the fired projectile hits with a higher speed.
2. To withstand “g-forces” of up to 10g’s, caused by suddenly pulling out of a steep dive, fighter jet pilots train on a “human centrifuge.” 10g is an acceleration of 98 m/s². If the length of the centrifuge arm is 12 m, at what speed is the rider moving when she experiences 10g’s?

Solution

The centripetal acceleration is just \( a_{\text{cent}} = \frac{v^2}{r} \), where \( v \) is the velocity, and \( r \) is the distance from the center of the “orbit.” So, solving for the velocity gives \( v = \sqrt{ar} \). Plugging in the numbers gives

\[
v = \sqrt{ar} = \sqrt{98 \times 12} = 34.3 \text{ m/s},
\]

which is about 77 miles per hour.
3. A wall clock has a minute hand with a length of 0.50 m and an hour hand with a length of 0.25 m. Take the center of the clock as the origin, and use a Cartesian coordinate system with the positive $x$ axis pointing to 3 o’clock and the positive $y$ axis pointing to 12 o’clock. Using unit vectors $\hat{i}$ and $\hat{j}$, express the position vectors of the tip of the hour hand ($\vec{A}$) and the tip of the minute hand ($\vec{B}$) when the clock reads (a) 12:00, (b) 3:00, (c) 6:00, (d) 9:00.

Solution

(a) Both hands point up (along the $y$ axis) at 12:00, and so $\vec{A} = 0.25\hat{j}$, and $\vec{B} = 0.5\hat{j}$.

(b) The hour hand points horizontally (along the $x$ axis), while the minute hand points up, so $\vec{A} = 0.25\hat{i}$, and $\vec{B} = 0.5\hat{j}$.

(c) The hour hand points down, while the minute hand points up, so $\vec{A} = -0.25\hat{j}$, and $\vec{B} = 0.5\hat{j}$.

(d) The hour hand points to the left, while the minute hand points up, so $\vec{A} = -0.25\hat{i}$, and $\vec{B} = 0.5\hat{j}$. 

4. A cat is chasing a mouse. The mouse runs in a straight line at a speed of 1.5 m/s. If the cat leaps off the floor at a 30° angle and a speed of 4.0 m/s, at what distance behind the mouse should the cat leap in order to land on the poor mouse?

**Solution**

We can picture the situation as seen in the figure to the right. The cat has a range of

\[ R(\theta) = \frac{v_0^2 \sin 2\theta}{g}. \]

For the given numbers, the cat jumps a distance

\[ R(\theta) = \frac{4^2 \sin 60^\circ}{9.8} = 1.41 \text{ m.} \]

It takes him a time

\[ T = \frac{2v_0 \sin \theta}{g} = \frac{8 \sin 30^\circ}{9.8} = 0.41 \text{ seconds}. \]

During this time, the mouse moves a distance

\[ d = v_{\text{mouse}} T = 1.5 \times 0.41 = 0.61 \text{ meters}. \]

So, in order for the cat to pounce on the mouse, he should start back a distance

\[ D = R - d = 1.41 - 0.61 = 0.80 \text{ meters, or 80 centimeters}. \]
5. Human blood contains plasma, platelets, and blood cells. To separate the plasma from other components, centrifugation is used. Effective centrifugation requires subjecting blood to an acceleration of 2000g or more. In this situation, assume that blood is contained in test tubes that are 15 cm long and are full of blood. These tubes ride in the centrifuge tilted at an angle of 45.0° above the horizontal.

(a) What is the distance of a sample of blood from the rotation axis of a centrifuge rotating at 3500 rpm, if it has an acceleration of 2000g?

(b) If the blood at the center of the tubes revolves around the rotation axis at the radius calculated in Part (a), calculate the accelerations experienced by the blood at each end of the test tube. Express all accelerations as multiples of $g$.

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**Solution**

(a) The acceleration that the blood experiences is the centripetal acceleration, $a_{\text{cent}} = \frac{v^2}{r}$. Now, the velocity is $v = 2\pi r / T$, where $T$ is the period. The period can be rewritten in terms of the frequency, $f = T^{-1}$, and so $v = 2\pi f r$. So, $a = 4\pi^2 f^2 r$. Since the centrifuge rotates at 3500 rpm, it makes $3500 / 60 = 58.3$ rotations per second, which is the frequency, $f$. Thus, solving for the distance, and plugging in the numbers gives

$$r = \frac{a}{4\pi^2 f^2} = \frac{2000 \times 9.8}{4\pi^2 \times 58.3^2} = 0.15 \text{ m} = 15 \text{ cm}.$$ 

This value seems large, but the centrifuge is spinning *really* fast!

(b) Because the test tube is tilted, the top and bottom of the tube are at different radii from the axis. This leads to different accelerations. Since the center of the test tube is at 15 centimeters, and is also 15 centimeters long, the top of the test tube is at $r_- = 15 - 7.5 \sin 45^\circ = 9.70$ cm. The bottom of the test tube is at $r_+ = 15 + 7.5 \sin 45^\circ = 20.3$ cm. The acceleration is, from before,

$$a_{\pm} = 4\pi^2 f^2 r_{\pm}.$$ 

Plugging in each value, and dividing by $g = 9.8$ to express the acceleration in terms of $g$, we find

$$\frac{a_-}{g} = \frac{4\pi^2 f^2}{g} r_- = \frac{4\pi^2 \cdot 58.3^2}{9.8} \cdot 0.097 = 1328$$

$$\frac{a_+}{g} = \frac{4\pi^2 f^2}{g} r_+ = \frac{4\pi^2 \cdot 58.3^2}{9.8} \cdot 0.203 = 2780$$

So, the accelerations range from about $1330g$, to about $2780g$. 

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6. Wile E. Coyote (*Carnivorus hungribilus*) is chasing the Roadrunner (*Speedibus cantcatchmi*) yet again. While running down the road, they come to a deep gorge, 15.0 m straight across and 100 m deep. The Roadrunner launches himself across the gorge at a launch angle of 15° above the horizontal, and lands with 1.5 m to spare.

(a) What was the Roadrunner’s launch speed?

(b) Wile E. Coyote launches himself across the gorge with the same initial speed, but at a different launch angle. To his horror, he is short of the other lip by 0.50 m. What was his launch angle? (Assume that it was less than 15°.)

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**Solution**

For a given launch speed and angle, the range of a projectile is given by

\[ R = \frac{v_0^2}{g} \sin(2\theta). \]

(a) The roadrunner jumps at an angle of \( \theta = 15^\circ \), and reaches a range \( R = 16.5 \) meters. So, solving for the velocity gives

\[ v_0 = \sqrt{\frac{Rg}{\sin(2\theta)}} = \sqrt{\frac{16.5 \times 9.8}{\sin 30^\circ}} = 18 \text{ m/s}. \]

(b) The poor coyote only jumps 14.5 meters. Solving for the angle gives

\[ \theta = \frac{1}{2} \sin^{-1} \left( \frac{Rg}{v_0^2} \right) = \frac{1}{2} \sin^{-1} \left( \frac{14.5 \times 9.8}{18^2} \right) = 13^\circ. \]
7. You are asked to consult for the city’s research hospital, where a group of doctors is investigating the bombardment of cancer tumors with high-energy ions. The ions are fired directly toward the center of the tumor at speeds of $5.0 \times 10^6$ m/s. To cover the entire tumor area, the ions are deflected sideways by passing them between two charged metal plates that accelerate the ions perpendicular to the direction of their initial motion. The acceleration region is 5.0 cm long, and the ends of the acceleration plates are 1.5 m from the patient. What acceleration is required to deflect an ion 2.0 cm to one side?

**Solution**

The plates add in a vertical acceleration, $a_y$. The horizontal acceleration is always zero, $a_x = 0$. The ion experiences a constant acceleration between the plates, and is deflected a height $H_1 = \frac{1}{2}a_y t_1^2$ where $t_1$ is the amount of time the ion spends in between the plates. Since the ion is traveling at a constant speed, $v_x$, then if the length of the plates is $d$, then $t_1 = d/v_x$. So, the net displacement from the plates is $H_1 = \frac{a_y d^2}{2v_x^2}$.

After emerging from the plates, the ion travels along at a constant speed, in a straight line. The ion then travels up a distance $H_2 = v_y t_2$ where $v_y$ is it’s velocity, while $t_2$ is the time the ion spends going between the plates to the target. If the distance away is $L$, then $t_2 = L/v_x$. Furthermore, the velocity is just what it’s picked up from being between the plates, $v_y = a_y t_1 = a_y d/v_x$. So, $H_2 = \frac{a_y dL}{v_x^2}$.

These two displacements need to add up to the $h = 2$ centimeter deflection. So, $h = H_1 + H_2 = \frac{a_y d^2}{2v_x^2} + \frac{a_y dL}{v_x^2}$. Solving for the acceleration gives

$$a_y = \frac{2hv_x^2}{d^2 + 2dL}.$$ 

Plugging in our numbers gives

$$a_y = \frac{2hv_x^2}{d^2 + 2dL} = \frac{2(0.02) (5 \times 10^6)^2}{0.05^2 + 2(0.05)(1.5)} = 6.56 \times 10^{12} \text{ m/s}^2,$$

which is a huge acceleration, roughly 100 billion times that due to gravity!
8. A toy cannon is placed on a ramp that has a slope of angle $\phi$. (a) If the cannonball is projected up the hill at an angle of $\theta_0$ above the horizontal and has a muzzle speed of $v_0$, show that the range $R$ of the cannonball (as measured along the ramp) is given by

$$R = \frac{2v_0^2 \cos^2 \theta_0 (\tan \theta_0 - \tan \phi)}{g \cos \phi}.$$ 

Ignore any effects due to air resistance.

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Solution

The path of the cannonball can be expressed in terms of the distance $x$ as

$$y(x) = (\tan \theta_0)x - \left( \frac{g}{2v_0^2 \cos^2 \theta_0} \right) x^2,$$

where we have set the initial $y_0 = 0$, since we set the cannon at the origin. Now, if the distance along the ramp is $R$, then the cannonball hits the ramp at $(x, y) = (R \cos \phi, R \sin \phi)$. Plugging in these values to the range expression gives

$$R \sin \phi = (\tan \theta_0)R \cos \phi - \left( \frac{g}{2v_0^2 \cos^2 \theta_0} \right) R^2 \cos^2 \phi,$$

or, dividing by $R \cos \phi$ gives

$$\tan \phi = \tan \theta_0 - \left( \frac{g}{2v_0^2 \cos^2 \theta_0} \right) R \cos \phi.$$

Now, rearranging and solving for $R$ gives

$$R \cos \phi = \frac{2v_0^2 \cos^2 \theta_0}{g} (\tan \theta_0 - \tan \phi),$$

or

$$R = \frac{2v_0^2 \cos^2 \theta_0}{g \cos \phi} (\tan \theta_0 - \tan \phi),$$

which is the answer we were looking for.