Course Description

The overarching goal of this course is to introduce the fundamentals of asymptotic analysis and perturbation methods for solving problems arising in the study of differential equations and integrals.

Instructor

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Lecture Time & Location

MWF, 11:30 am – 12:20 pm, COB1 205

Discussion Time & Location

R, 11:30 am – 12:20 pm, COB2 262

Office hours

TBA

Learning outcomes. By the end of this course, you should be able to:

1. Have a working knowledge of infinite series solutions to differential equations.
2. Recognize the advantages and limitations of asymptotic approximations to challenging mathematical problems.
3. Compute asymptotic approximation of integrals.
4. Compute perturbation solutions to differential equations.
5. Combine numerical and asymptotic solutions to differential equations.

Relationship to Program Learning outcomes. This course will contribute to teach you to:

1. PLO #1. Solve advanced mathematical problems using analytical methods (in class and homework).
2. PLO #2. Solve advanced mathematical problems using computational methods (in homework).
3. PLO #3. Give clear and organized written explanations of mathematical ideas (in homework).

In particular, you will learn to combine analytical and numerical skills to provide a comprehensive description of solutions to mathematical problems.

Prerequisites

A course on ordinary differential equations with boundary value problems and a course on complex variables are *strongly encouraged*.

Topics covered: There are three main topics in this course

1. Power series solutions to differential equations about
   (a) an ordinary point
   (b) a regular singular point (Frobenius' method)
   (c) an irregular singular point (Fuchs' method)
2. Asymptotic methods for approximations of integrals
   (a) Integration by parts
   (b) Laplace's method and Watson's Lemma
   (c) Method of stationary phase
   (d) Method of steepest descent
3. Perturbations methods for differential equations
   (a) Non-dimensionalization and scaling
   (b) Regular perturbation expansions
   (c) Singular perturbation expansions: boundary layer theory, WKB, Method of Multiple Scales

Textbook. The required textbook is *Advanced Mathematical Methods for Scientists and Engineers*, by Bender and Orszag (Springer-Verlag, New York, 1999), a copy of which is available at the UC Merced Library. In addition, you may want to consult the following textbooks.

1. *Complex Variables* by Ablowitz and Fokas (copy available at the UC Merced library),
2. *Asymptotic Expansions of Integrals* by Bleistein and Handelsman (copy available at the UC Merced library),
3. *Elementary Differential Equations* by Boyce and DiPrima (copy available at the University of California library),
4. *Ordinary Differential Equations* by Ince (copy available at the University of California library),
5. *Advanced Calculus for Applications* by Hildebrand (copy available at the UC Merced library),
6. *Perturbation Methods* by Hinch (copy available at the UC Merced library),
7. *Multiple Scale and Singular Perturbation Methods* by Kevorkian and Cole (copy available at the UC Merced library).

Course webpage. The Math 223 website is part of the CatCourses system.

Homework. There will be approximately bi-weekly homework assignments throughout the semester. You may work on the homework in pairs, but each student must turn in his or her own individual work.

Exams. There will be a take-home final exam at the end of the semester. No midterm exam will be administered.

Grade determination. Your grade will be determined based on your homework assignments (50%), your participation in class (10%) and your final exam score (40%). Letter grades will not correspond strictly to

any fixed numerical scale, but rather loosely correspond to:
A indicates that you have understood every major concept in the class
B indicates that you have understood most major concepts in the class
C indicates that you have understood only some of the major concepts in the class
D indicates that you have no real understanding of most of the major concepts in the class
F indicates that you have gained virtually no understanding in this class.

Cell phones, calculators and computers. No such device is necessary or allowed in the lectures.

Extra help. You are encouraged to get extra help whenever you need it, by coming to office hours, or working in small groups. You are welcome to send questions to your instructor via e-mail at any time.

Dropping the Course

Please see the UC Merced General Catalog for more details.

Special accommodations

Student Affairs determines accommodations based on documented disabilities. If you qualify, please submit a letter from Disability Services to the instructor; every effort will be made to accommodate your needs.

I will also make every effort to accommodate students whose religious beliefs/obligations lead to scheduling conflicts with exams, assignments, or attendance. Please speak with me during the first two weeks of this course regarding any potential accommodations.

Academic integrity

Academic integrity is the foundation of an academic community and without it none of the educational or research goals of the university can be achieved. All members of the university community are responsible for its academic integrity. Existing policies forbid cheating on examinations, plagiarism and other forms of academic dishonesty. The current policies for UC Merced are described in the Academic Honesty Policy. See http://studentlife.ucmerced.edu/ $\rightarrow$ "Student Judicial Affairs"