Parabolic Pulse Generation in Gain-Guided Optical Fibers with Nonlinearity

Chenji Gu, Boaz Ilan, and Jay E. Sharping
University of California - Merced, Merced, CA 95344 USA (phone: 209-228-4049; fax: 209-228-4053;
e-mail: jsharping@ ucmerced.edu).

Abstract — **We explore the combined effects of nonlinearity and gain-guidance on the propagation and amplification of high power parabolic pulses within fibers. By choosing a suitable gain coefficient, group-velocity dispersion profile, and fiber size one can increase output power and reduce the time-bandwidth product of pulses.**

I. Introduction

In this submission we show that if the Kerr nonlinearity is added to the model for a Gain-Guided optical Fiber (GGF) amplifier then self-similar parabolic pulse propagation can be realized. We also highlight the benefits of using GGF instead of fiber amplifiers. In the presence of normal group velocity dispersion one obtains parabolic pulses or similaritons which propagate with a perfect linear chirp. The output of such a GGF amplifier can be of higher peak power, and reduced time-bandwidth product compared with conventional fiber amplifiers. To date only linear-optical processes in GGF have been considered; but since these novel fibers are potentially useful for high-power optical amplifiers and oscillators, it is essential to study the limitations and opportunities that nonlinear-optical effects present.

The GGF, as proposed by A. E. Siegman, confines light through a combination of index contrast and gain [1, 2]. One of the novelties of these fibers is that they can support a guided mode in the core region even if the refractive index contrast by itself leads to leaky or "index anti-guided" (IAG) behavior. Subsequent experimental measurements from other groups [3-5] confirm many of Siegman's predictions. The first measurements were performed in a laser configuration using a heavily Nd-doped optical fiber with a 100-300 μm core and with a core-to-cladding index contrast less than one, which would usually lead to leaky propagation. In the presence of gain, however, output beam profile measurements reveal the transition from an unguided to a guided mode when the gain is increased beyond a certain threshold.

II. Theory and Simulation

We present a dynamical model of a GGF with inclusion of the Kerr nonlinearity and explore pulse propagation according to this model. Within the waveguide the complex index of refraction has both a real part owing to linear and nonlinear phase-velocity change and an imaginary part owing to the gain. The standard representation of a GGF is show in Fig. 1, where an amplitude growth coefficient, *g,* having the same uniform distribution as the index step, Δn (for IAG*)*, is included within the core region.

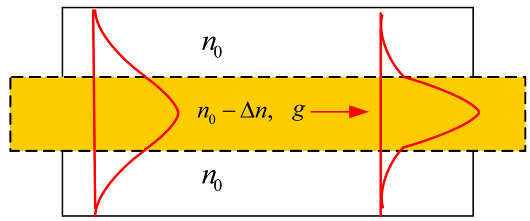

Fig. 1 Gain guiding plus index anti-guiding, with a negative index step: $-\Delta$n in the core

The model for gain-guidance is based on the scalar wave equation in the medium with nonlinear electric polarization which is assumed to be small compared to the linear polarization. Adding the gain to the imaginary part of the refractive-index change and the anti-index core to the real part gives to leading order [7], and we obtain

$$\begin{aligned}\mathbf{D} &= \varepsilon\mathbf{E} + \mathbf{P}_{NL} \\ &= \varepsilon_0\left(n - \Delta n + n_2|E|^2 + j(\lambda/2\pi)g\right)^2\mathbf{E} \\ &\cong \varepsilon_0 n^2\mathbf{E} + \varepsilon_0 j(\lambda/2\pi)g\mathbf{E} + 2\varepsilon_0 n n_2|E|^2\,\mathbf{E},\end{aligned} \tag{1}$$

where the approximation is based on the core has a slightly lower refractive index than the cladding, experimentally around 0.35% [5]. Although the gain-guiding effect is quite weak it becomes meaningful for large gain coefficients (g=1 m^{-1}) corresponding to an imaginary index of only around 10^{-5} [5]. This evidence suggest that first-order perturbation theory is good enough for the derivation of pulse prorogation equation [6].

Assuming gain-guidance is achieved, pulse propagation is modeled as nonlinear propagation with gain. The nonlinear Schrödinger equation (NLSE) with gain [8] is given by:

$$j\frac{\partial\Psi}{\partial z} = \frac{1}{2}\beta_2\frac{\partial^2\Psi}{\partial T^2} - \gamma|\Psi|^2\Psi + j\frac{g}{2}\Psi, \tag{2}$$

where $\Psi(z,T)$ is the slowly varying pulse envelope in the moving frame of the envelope group velocity, β_2 is the fiber's second order dispersion coefficient, γ is the Kerr nonlinear coefficient, and g is the gain coefficient. Mathematically, the model for a GGF with nonlinearity is equivalent to the NLSE with gain which is the standard expression for a fiber amplifier. The only difference is that the gain coefficient of a GGF is quite large.

It is well known that parabolic pulses or similaritons, i.e. the pulse is always a scaled version of itself, can be generated within fiber amplifiers [9, 10] in the normal dispersion domain

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(β_2>0). Our analysis above suggests that similariton evolution is likely to occur in GGF amplifiers as well. To see this, we use the asymptotic solution of Eq. (2), where the pulse amplitude A, temporal phase, Φ, and temporal width T_p is given by [10]:

$$\Psi(z,T) = A(z,T)\exp\left(j\Phi(z,T)\right) \tag{3}$$

$$A(z,T) = A_0 \exp\left(\frac{g}{3}z\right)\sqrt{1-\frac{T^2}{T_p^2(z)}} \tag{4}$$

$$\Phi(z,T) = \varphi_0 + \frac{3\gamma A_0^2}{2g}\exp\left(\frac{2}{3}gz\right) - \frac{g}{6\beta_2}T^2, \tag{5}$$

where U_{in} is the input pulse energy, $A_0 = \frac{1}{2}\left(\frac{gU_{in}}{\sqrt{\gamma\beta_2/2}}\right)^{1/3}$ characterizes the amplitude of the pulse, and $T_p = \frac{6\sqrt{\gamma\beta_2/2}}{g}A_0\exp\left(\frac{g}{3}z\right)$ characterizes its width. The solution above is valid for large propagation distances for $|T| \le T_p(z)$. More precisely, the dimensionless product gz is assumed to be large. It is this product that determines the rate at which parabolic pulses spread.

We now compare between the evolution of parabolic pulses in a GGF amplifier with those in standard fiber amplifier. Suppose all the parameters are identical for the two cases except the gain coefficient: 100-pJ input pulses with β_2=+25ps^{2}m^{-1}, γ=5W^{-1}m^{-1}. For a GGF, g=100 m^{-1} [3]; whereas for a fiber amplifier g=1 m^{-1} [10]. In both cases the input pulse is a 1.4-ps hyperbolic secant. Figure 2 shows the propagation of the pulses as they evolve: in the 10 m fiber amplifier and in a 10cm GGF aimplifier, respectively. The length of the two fibers was chosen to keep the gain-distance product constant, i.e., $gz = 1$.

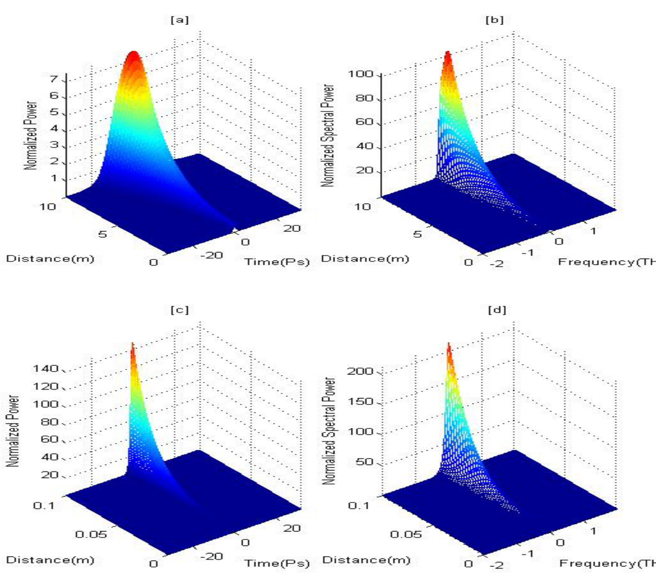

Fig. 2 Similariton evolution in a 10-m long GGF (a) time domain (b) frequency domain. Similariton evolution in a 10-cm long erbium fiber amplifer (c) time domain (d) frequency domain.

From Fig. 2, we can find two major advantages of using GGF to generate similaritons: much higher center peak power and narrower pulses in both time and frequency domain. These conclusions are also evident from Eqs. (3)- (5) for the asymptotic solution. The gain coefficient, g, is in the denominator of the expression for T_p; and amplitude A_0 is proportion to $g^{1/3}$. If the gain is a factor of 100 times bigger, the output of peak power of the pulse will be $100^{2/3}$=20 times higher while the output pulse width will also be $100^{2/3}$=20 times shorter. Furthermore, these two features of similariton propagation in nonlinear GGF may have applications in mode-locked similariton lasers [11] as well as future optical similariton communications [12].

Despite the use of short fibers for GGF amplifiers, it appears that parabolic pulses can be generated. The parameters in NLSE with gain Eq. (2) are dependent on the fiber material and structure. Successful demonstration of similariton propagation in GGF will depend on simultaneously satisfying the need for a large gain coefficient with suitable nonlinearity. The existing GGF systems have relied on a laser configuration and are not that suitable for similariton generation as the nonlinearity is too small owing to the very large fiber core [4]. However, simply making the core smaller will also decrease the gain coefficient, which is critical to the threshold of lasing [4].

In summary, we present the combination of gain guidance and parabolic pulse propagation as a method to achieve high power parabolic pulses. Simulation results show that using a few centimeters of GGF one can realize parabolic pulse generation for reasonable values of nonlinearity, gain coefficient, and chromatic dispersion for GGF.

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