Deterministic vectorial effects lead to multiple filamentation

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The standard explanation for multiple filamentation of laser pulses is that it is caused by noise in the input beam. We propose an alternative explanation that is based on deterministic vectorial (polarization) effects. We present numerical simulations in support of the vectorial-effects explanation and suggest a simple experiment for deciding whether multiple filamentation is due to vectorial effects. © 2001 Optical Society of America

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The propagation of intense laser beams in a Kerr medium is governed by the vector Helmholtz equations[1]

$$\Delta \boldsymbol{\mathcal{E}} - \nabla(\nabla \cdot \boldsymbol{\mathcal{E}}) + {k_0}^2 \boldsymbol{\mathcal{E}} = -\frac{{k_0}^2}{\epsilon_0 {n_0}^2} \boldsymbol{\mathcal{P}}_{\rm NL}\,,$$

$$\nabla \cdot \boldsymbol{\mathcal{E}} = -\frac{1}{\epsilon_0 {n_0}^2} \nabla \cdot \boldsymbol{\mathcal{P}}_{\rm NL}\,. \qquad (1)$$

Here, $\boldsymbol{\mathcal{E}} = (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3)$ is the electric field in the (x, y, z) directions, respectively, ϵ_0 is vacuum permittivity, n_0 is the medium's (linear) refractive index, k_0 is the wave number, and $\Delta = \partial_{xx} + \partial_{yy} + \partial_{zz}$. The nonlinear polarization vector $\mathbf{P}_{\rm NL}$ in an isotropic Kerr medium is given by[1,2]

$$\mathbf{P}_{\rm NL}(\boldsymbol{\mathcal{E}}) = \frac{4\epsilon_0 n_0 \overline{n}_2}{1+\gamma} (|\boldsymbol{\mathcal{E}}|^2 \boldsymbol{\mathcal{E}} + \gamma \boldsymbol{\mathcal{E}}^2 \boldsymbol{\mathcal{E}}^*)\,, \qquad (2)$$

where $|\boldsymbol{\mathcal{E}}|^2 := \sum_{i=1}^3 |\mathcal{E}_i|^2$, $\boldsymbol{\mathcal{E}}^2 := \sum_{i=1}^3 {\mathcal{E}_i}^2$, $\boldsymbol{\mathcal{E}}^*$ is the complex conjugate of $\boldsymbol{\mathcal{E}}$, $\overline{n}_2$ is the Kerr coefficient, and γ is a positive constant whose value depends on the physical origin of the Kerr effect.[3]

Let us set the coordinate system such that the laser beam is linearly polarized in the x direction and propagates in the positive z direction as it enters the Kerr medium at $z = 0$. Almost all studies ignore the vectorial nature of the beam and assume that the beam remains linearly polarized inside the Kerr medium, i.e., $\mathcal{E}_2 = \mathcal{E}_3 = 0$ for $z \geq 0$. In that case, the Kerr effect [Eq. (2)] is described by the scalar relation $n = n_0 + \overline{n}_2 |\mathcal{E}_1|^2$ and, to leading order, Eqs. (1) and (2) reduce to the nonlinear Schrödinger equation (NLS) for the beam amplitude $\mathcal{A}_1$:

$$2ik_0 \mathcal{A}_{1,z} + \Delta_\perp \mathcal{A}_1 + \frac{2{k_0}^2 \overline{n}_2}{n_0} |\mathcal{A}_1|^2 \mathcal{A}_1 = 0\,, \qquad (3)$$

where $\mathcal{E}_1 = \mathcal{A}_1(z, x, y)\exp(ik_0 z)$ and $\Delta_\perp = \partial_{xx} + \partial_{yy}$.

The NLS model [Eq. (3)] has been successful in predicting the catastrophic self-focusing of intense laser beams[4] whose input power P_0 is above the critical power for collapse P_c.[5] When $P_0 \gg P_c$, catastrophic self-focusing is often preceded by multiple filamentation, in which a single input beam breaks up into several long and narrow filaments.[6] Because, according to the NLS model, beams with axially symmetric input profiles should remain axially symmetric during propagation, the question arises as to what the mechanism is behind the breakup of axial symmetry that leads to multiple filamentation. For more than 30 years, the standard (and only) explanation for multiple filamentation in isotropic homogeneous media, which is due to Bespalov and Talanov,[7] has been that breakup of axial symmetry is initiated by random noise in the input beam's profile. However, the validity of the analysis in Ref. 7 is questionable, because it is based on stability analysis of plane-wave solutions. In this Letter we propose an alternative, deterministic explanation for multiple filamentation that is based on vectorial effects. We present numerical simulations in support of the vectorial-effects explanation and suggest a simple experiment for deciding whether multiple filamentation is due to vectorial effects.

We begin by deriving a new scalar partial differential equation for self-focusing in the presence of vectorial effects. We nondimensionalize Eqs. (1) and (2), using $\tilde{x} = x/r_0$, $\tilde{y} = y/r_0$, $\tilde{z} = z/2k_0{r_0}^2$, and $\boldsymbol{\mathcal{E}} = (2r_0k_0)^{-1}\sqrt{n_0/\overline{n}_2}\,\mathbf{A}(z, x, y)\exp(ik_0 z)$, where r_0 is the initial beam width. Dropping the tildes, we get the nondimensional vectorial system

$$iA_{,z} + \Delta_\perp \mathbf{A} + {}^1\!/\!_4 f^2 \mathbf{A}_{,zz} + \mathbf{N} = -[f\boldsymbol{\nabla}_\perp + \hat{e}_3(i + {}^1\!/\!_2 f^2 \partial_z)] \times (f\boldsymbol{\nabla}_\perp \cdot \mathbf{N} + iN_3 + {}^1\!/\!_2 f^2 N_{3,z})\,,$$

$$f\boldsymbol{\nabla}_\perp \cdot \mathbf{A} + iA_3 + {}^1\!/\!_2 f^2 A_{3,z} = -4f^2(f\boldsymbol{\nabla}_\perp \cdot \mathbf{N} + iN_3 + {}^1\!/\!_2 f^2 N_{3,z})\,, \qquad (4)$$

where $\mathbf{N}(\mathbf{A}) = (|\mathbf{A}|^2\mathbf{A} + \gamma \mathbf{A}^2 \mathbf{A}^*)/(1+\gamma)$, $f = (r_0k_0)^{-1}$, $\hat{e}_3 = (0, 0, 1)$, and $\boldsymbol{\nabla}_\perp = (\partial_x, \partial_y, 0)$. Because $f = \lambda/2\pi r_0 << 1$, we can use perturbation analysis to show that $A_3 \sim ifA_{1,x}$ and $A_2/A_1 = O(f^2)$. Therefore the system of Eqs. (4) can be reduced to the following scalar equation for A_1 (Ref. 8):

$$iA_{1,z} + \Delta_\perp A_1 + |A_1|^2 A_1 = -f^2\left[\frac{1}{4} A_{1,zz} + \frac{4+6\gamma}{1+\gamma} |A_{1,x}|^2 A_1 + (A_{1,x})^2 {A_1}^* + \frac{1+2\gamma}{1+\gamma} (|A_1|^2 A_{1,xx} + {A_1}^2 {A_{1,xx}}^*)\right], \qquad (5)$$

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where $A_{1,zz}$ is beam nonparaxiality and the other terms on the right-hand side of Eq. (5) correspond to vectorial effects.[9] We remark that previous studies of vectorial effects[10–13] obtained similar, yet not identical, equations for A_1. Clearly, when $f = 0$, Eq. (5) reduces to the nondimensional NLS:

$$iA_{1,z} + \Delta_\perp A_1 + |A_1|^2 A_1 = 0. \quad (6)$$

The asymmetry in the x and y derivatives in Eq. (5) implies that vectorial effects breakup the axial symmetry while inducing a preferred direction in the transverse plane (the direction of input beam polarization). Indeed, axial symmetry is broken at the vectorial model [Eq. (1)] by the linear polarization of the input beam. In this Letter we show that the breakup of axial symmetry by vectorial effects can lead to multiple filamentation. A typical simulation can be seen in Fig. 1, where we solve Eq. (5) for an axially symmetric Gaussian input beam with $P_0 = 5P_c$. As the beam propagates, it goes through the following stages: (i) nonaxial self-focusing, (ii) defocusing into a ring with two peaks, (iii) a second self-focusing, (iv) defocusing of the central peak and emergence of two filaments, and (v) self-focusing of the two filaments. During the last stage the two filaments propagate forward in the z direction while they move away from each other along the axis of initial polarization at a constant speed (Fig. 2).

Also, we numerically tested the Bespalov–Talanov model for multiple filamentation[7] by solving the unperturbed NLS (6) with high-power ($P_0 >> P_c$) axially symmetric input Gaussian beams, to which we added random noise, both in amplitude and in phase. We ran many simulations but could not see any evidence of multiple filamentation caused by random noise.[14] Rather, the beams converged to an axially symmetric profile as they collapsed. We also note that the Bespalov–Talanov model cannot explain the multiple filamentation experiments reported in Ref. 15 because "although [the filament patterns were] random in appearance, [they] were perfectly reproducible shot to shot."[15] We believe that the major weakness of the Bespalov–Talanov model is that it assumes that, to leading order, the electric field is a plane wave. Under this assumption (which implies infinite input power), instabilities can grow while the leading-order solution remains unchanged. Such is not the case, however, for a propagating beam, for which the transverse self-focusing dynamics of the leading-order solution dominate the evolution of the noise.

The results presented so far show that vectorial effects can lead to multiple filamentation for beams whose power is only a few times the critical power. They do not, however, rule out the possibility that under certain conditions multiple filamentation can result from noise in the input beam. Indeed, multiple filamentation has been observed in simulations of NLSs with saturating nonlinearity for very powerful beams ($P_0 \sim 100\ P_c$).[16–19] In addition, our model does not include other mechanisms, such as time dispersion, plasma generation, and photon absorption, which, in theory, could also lead to multiple filamentation. Nevertheless, we can propose a simple experimental test for deciding whether multiple filamentation results from vectorial effects. This test is based on the observation that vectorial effects are the only mechanism (neglected in the derivation of

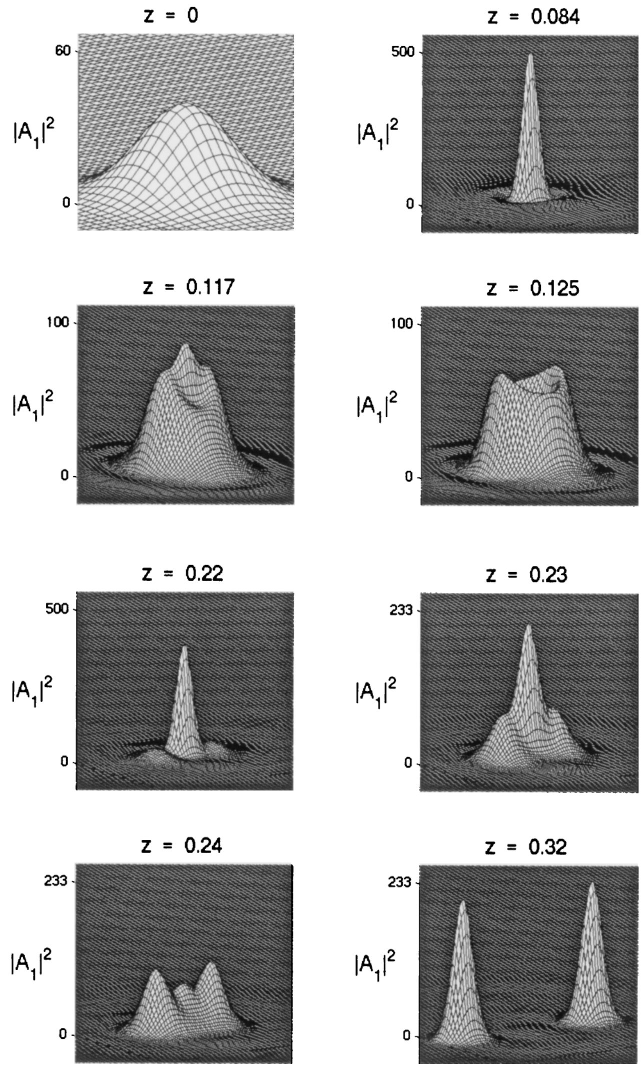

Fig. 1. Deterministic multiple filamentation of an axially symmetric input beam $A_1(z = 0, x, y) = 2\sqrt{5P_c}\ \exp[-(x^2 + y^2)]$. Here $f = 0.025$ and $\gamma = 0.5$.

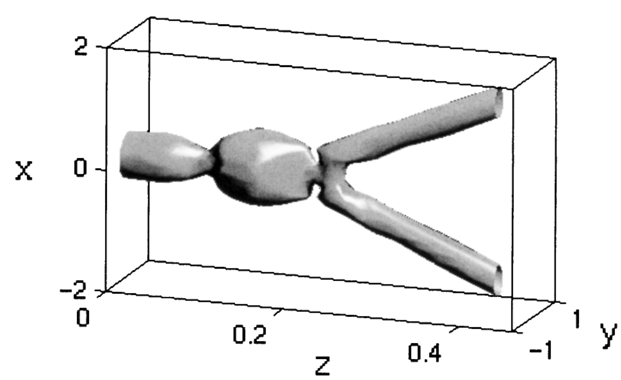

Fig. 2. Isosurface $|A_1|^2 \equiv 28$ of the data in Fig. 1.

the NLS model) that breaks up the axial symmetry while including a preferred direction in the transverse plane, which is that of the initial linear polarization. Therefore, if multiple filamentation results from vectorial effects, then (i) the filamentation pattern should persist between experiments (as reported in Ref. 15), (ii) if the direction of linear polarization of the input beam is rotated in the transverse plane between experiments, the filamentation pattern should follow the same rotation, and (iii) when a beam splits into two filaments, the splitting should occur either in the direction of initial polarization or perpendicular to it. If, in contrast, multiple filamentation results from noise in the input beam, the filamentation pattern should vary between experiments and, in particular, be independent of the direction of initial polarization. We note that this test can be applied to any filamentation experiment and not only to those governed by Eq. (1).

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